PARTICLE PRODUCTION OF COHERENTLY OSCILLATING NONCLASSICAL INFLATON IN FRW UNIVERSE

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Abstract

We study particle production of coherently oscillating inflaton in the semiclassical theory of gravity by representing inflaton in coherent and squeezed state formalisms. A comparative study of the inflaton in classical gravity with coherent state inflaton in semiclassical gravity is also presented.

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1 Introduction

The hot big bang model or standard cosmology is spectacularly successful. In short, it provides reliable and tested account of the history of the universe from about 0.01sec after the big bang until today, some 15 billion years later. Despite its success, the hot big bang model left many features of the universe unexplained. The most important of these are horizon problem, singularity problem, flatness problem, homogeneity problem, structure formation problem, monopole problem and so on. All these problems are very difficult and defy solution within the standard cosmology. Most of these problems have, in the past couple of decades, been either completely resolved or considerably relaxed in the context of one complete scenario, called inflationary scenario [1]. At present there are different versions [2-5] of the inflationary scenario. The main feature of all these versions is known as the inflationary paradigm. According to the simplest version of the inflationary scenario, the universe in the past expanded almost exponentially with time, while its energy density was dominated by the effective potential energy density of a scalar field, called the inflaton. Sooner or later, inflation terminated and the inflaton field started quasiperiodic motion with slowly decreasing amplitude. The universe was empty of particles after inflation and particles of various kinds created due to the quasiperiodic evolution of the inflaton field. The universe became hot again due the oscillations and decay of the created particles of various kinds. Form on ,it can be described by the hot big bang theory.

Most of the inflationary scenarios are based on the classical gravity of the Friedmann equation and the scalar field equation in the Friedmann-Robertson-Walker (FRW) universe, assuming its validity even at the very early stage of the universe. However, quantum effects of matter fields and quantum fluctuations are expected to play a significant role in this regime, though quantum gravity effects are still negligible. Therefore, the proper description of a cosmological model can be studied in terms of the semiclassical gravity of the semiclassical Friedmann equation with quantized matter fields as the source of gravity. Recently , the study of quantum properties of inflaton has been received much attention in semiclassical theory of gravity and inflationary scenarios. [6,7]. In the new inflation scenario [8] quantum effects of the inflaton were partially taken into account by using one-loop effective potential and an initial thermal condition. In the stochastic inflation [9] scenario the inflaton was studied quantum mechanically by dealing with the phase-space quantum distribution function and the probability distribution [10]. The semiclassical quantum gravity seems to be a viable method throughout the whole non-equilibrium quantum process from the pre-inflation period of hot plasma in thermal equilibrium to the inflation period and finally to the matter-dominated period.

The aforementioned studies show that results obtained in classical gravity are quite different from those in semiclassical gravity. Though both classical and quantum inflaton in the oscillatory phase of the inflaton lead the same power law expansion, the correction to the expansion does not show any oscillatory behavior in semiclassical gravity in contrast to the oscillatory behavior seen in classical gravity. It is to be noted that, the coherently

oscillating inflaton suffers from particle production. Such studies reveal that quantum effects and quantum phenomena play an important role in inflation scenario and the related issues. Recently, it has been found that nonclassical state formalisms are quite useful to deal with quantum effects in cosmology [11-21].

The goal of the present paper is to study a massive, minimal inflaton in the FRW universe in the context of semiclassical gravity by representing the scalar field in squeezed and coherent states. We study the particle production, of the oscillatory phase, of the inflaton in coherent and squeezed state formalisms in the semiclassical theory of gravity. We give a brief comparative study of the inflaton in classical gravity with the coherent state inflaton in semiclassical gravity.

2 Scalar field in FRW metric

Consider a flat Friedmann-Robertson-Walker spacetime with the line element (with c=1)

$$ds^2 = -dt^2 + S^2(t)(dx^2 + dy^2 + dz^2), \tag{1}$$

where $S(t)$ is an unspecified positive function of t. One can consider, the universe with such a metric, as an expanding universe, although $S(t)$ need not be increasing with time. The metric is treated as an unquantized external field.

The equation governing the scalar field is the Klein-Gordon equation [22]:

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2)\phi = 0 \ , \tag{2}$$

where $g^{\mu\nu}$ is the inverse of the metric tensor $g_{\mu\nu}$, ∇_μ is the covariant derivative and μ, ν = 0, 1, 2, 3 .

Assume, minimal coupling between the gravity and the scalar field, the massive scalar field dynamics is governed by the Lagrangian density:

$$\mathcal{L} = -\frac{1}{2}\sqrt{(-g)}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2) \ , \tag{3}$$

Using the metric (1), (3) becomes

$$L = \frac{1}{2}S^3(t)\left(\dot{\phi}^2 - S^{-2}(t)\sum_{j=1}^{3}(\partial_j\phi)^2 - m^2\phi^2\right) \tag{4}$$

and (2) becomes

$$\ddot{\phi} + 3\frac{\dot{S}(t)}{S(t)}\dot{\phi} + m^2\phi - \sum_{j=1}^{3}\partial_j^2\phi = 0, \tag{5}$$

where overdot represents a derivative with respect to time and $\partial_j \equiv \frac{\partial}{\partial x_j}$ represents the spatial derivative. The scalar field can be quantized [22] by defining momentum conjugate to ϕ as

$$\pi = \frac{\partial L}{\partial\dot{\phi}} \tag{6}$$

and following the canonical quantization procedure. The Hamiltonian of the scalar field can be obtained by using

$$H = \pi\dot{\phi} - L \ . \tag{7}$$

Therefore, the Hamiltonian of the scalar field is :

$$H = \frac{\pi^2}{2S^3(t)} + \frac{1}{2}S(t)\sum_{j=1}^{3}(\partial_j\phi)^2 + \frac{1}{2}S^3(t)m^2\phi^2 \ . \tag{8}$$

In the present study, we consider only the homogeneous models of the scalar field. For a homogeneous scalar field (inflaton), the spatial derivatives of ϕ vanish, hence the Hamiltonian of the inflaton can be written as,

$$H = \frac{\pi^2}{2S^3(t)} + \frac{1}{2}S^3(t)m^2\phi^2 \ . \tag{9}$$

Therefore, the temporal component of the energy-momentum tensor for the inflaton is obtained as

$$T_{00} = S^3(t)(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2) \ . \tag{10}$$

We, next discuss the physical and mathematical properties of the coherent and squeezed state formalisms briefly.

3 Coherent states and squeezed states

Squeezed states and coherent states are important classes of quantum states, well-known in quantum optics [23]. At a glance, it may appear that coherent and squeezed states formalisms and cosmology are two different branches of physics having no connections. However, the mathematical and physical properties of these states find much use in the study of many issues in cosmology. Recently, squeezed and coherent states are being used as probes for studying the quantum effects in cosmology such as cosmological particle creation [12], entropy generation [18], detection of gravitational waves [24], inflationary scenario [17] etc. Coherent states are considered as most classical, that can be generated from the vacuum state $\mid 0\rangle$ by the action of displacement operator. In the present study, we use single mode coherent and squeezed states only. A single mode coherent state can be defined [25] as

$$\mid \alpha\rangle = D(\alpha) \mid 0\rangle \ , \tag{11}$$

where $D(\alpha)$ is the single mode displacement operator, given by

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \ . \tag{12}$$

Here, α is a complex number and a, $a^\dagger$ are respectively the annihilation and creation operators, satisfying $[a, a^\dagger] = 1$. The action of a and $a^\dagger$ on the coherent state gives

$$\begin{aligned} a \mid \alpha\rangle &= \alpha \mid \alpha\rangle \\ a^\dagger \mid \alpha\rangle &= \alpha^* \mid \alpha\rangle \ . \end{aligned} \tag{13}$$

The single mode displacement operator given by (12) satisfy the following properties.

$$\begin{aligned} D^\dagger a D = &\quad a+\alpha \\ D^\dagger a^\dagger D = &\quad a^\dagger + \alpha^* \ . \end{aligned} \tag{14}$$

A squeezed state is generated by the action of the squeezing operator on any coherent state is also on the vacuum state. Therefore, a single mode squeezed state is defined [25] as

$$\mid \alpha, \xi\rangle = Z(r,\varphi) D(\alpha)|0\rangle \ , \tag{15}$$

with $Z(r,\varphi)$, the single mode squeezing operator, given by,

$$Z(r,\varphi) = \exp \frac{r}{2}(e^{-i\varphi}a^2 - e^{i\varphi}a^{\dagger 2}) \ . \tag{16}$$

Here, r is the squeezing parameter, which determines the strength of the squeezing and φ is the squeezing angle, which determines the distribution between conjugate variables, with $0 \leq r \leq \infty$ and $-\pi \leq \varphi \leq \pi$. The squeezing operator satisfy the following properties

$$\begin{aligned} Z^\dagger a Z = &\quad a\cosh r - a^\dagger e^{i\varphi}\sinh r \\ Z^\dagger a^\dagger Z = &\quad a^\dagger \cosh r - a e^{-i\varphi}\sinh r \ . \end{aligned} \tag{17}$$

By setting α=0 in (15), one obtains the squeezed vacuum state, and is defined [25] as:

$$\mid \xi\rangle = Z(r,\varphi) \mid 0\rangle \ . \tag{18}$$

The squeezed vacuum state is a many-particle state and hence the resulting field may be called classical. However, the statistical properties of these states greatly differ from the coherent states and therefore, this state is considered as highly non-classical having no analog in classical physics. In the case of coherent states, the variance of the conjugate variables are always equal to each other, while in a squeezed state one component of the noise is always squeezed with respect to the other. Therefore, in (x,p) plane, the noise for the coherent state can be described by a circle and for the squeezed state as an ellipse.

4 Particle production of nonclassical inflaton in semi-classical gravity

In the quantum theory of a matter field in curved space-time, the background metric is usually treated as classical while the matter field is treated as quantum. Two standard approaches towards the quantum field have been developed; one is the conventional field theoretical approach, the other being the canonical quantum gravity. The correct theory of a quantum fluctuating geometry and matter field has not yet been completely developed; it would be meaningful to consider the semiclassical gravity theory to study the quantum effect of matter field in a prescribed background metric. The semiclassical approach is also

useful to deal with problems in cosmology, where quantum gravity effects are negligible. In the present study, we consider the oscillatory phase of inflaton after the inflation, where quantum gravity effects are negligible. Therefore the present study can be restricted in the frame work of semiclassical gravity. In semiclassical theory the Einstein equation can be written as (here onwards we use the unit system $c = \hbar$=1 and $G = \frac{1}{m_p^2}$):

$$G_{\mu\nu} = \frac{8\pi}{m_p^2} \langle \hat{T}_{\mu\nu} \rangle \ . \tag{19}$$

The source of the gravitational field is the quantum , a scalar field ϕ, governed by the time-dependent Schr$\ddot{o}$dinger equation,

$$i \frac{\partial}{\partial t} \Phi(\phi, t) = \hat{H}_m(\phi, t) \Phi(\phi, t) \ . \tag{20}$$

Here, $\hat{H}_m$ represents the Hamiltonian for the matter field.

As mentioned earlier, we consider a massive inflaton, minimally coupled to a spatially flat FRW universe with the metric (1). Therefore, the time-time component of the classical gravity is now the classical Einstein (or Friedmann) equation

$$\left(\frac{\dot{S}}{S} \right)^2 = \frac{8\pi}{3m_p^2} \frac{T_{00}}{S^3(t)} \ , \tag{21}$$

where T_{00} is the energy density of the inflaton, given by (10). The classical equation of motion for the inflaton is obtained from (5):

$$\ddot{\phi} + 3 \frac{\dot{S}(t)}{S(t)} \dot{\phi} + m^2 \phi = 0 \ . \tag{22}$$

In the cosmological context, the classical Einstein equation (21) means that the Hubble constant, $H = \frac{\dot{S}}{S}$, is determined by the energy density of the dynamically evolving inflaton as described by (22). In the semiclassical theory, the Friedmann equation can be written as:

$$\left(\frac{\dot{S}}{S} \right)^2 = \frac{8\pi}{3m_p^2} \frac{1}{S^3(t)} \langle \hat{H}_m \rangle \ , \tag{23}$$

where $\langle \hat{H}_m \rangle$ represent the expectation value of the Hamiltonian of the scalar field in a quantum state under consideration.

Consider, a massive inflaton, minimally coupled to the spatially flat FRW metric . The inflaton can be described by the time dependent harmonic oscillator, with the Hamiltonian given in (9). To study, the semiclassical Friedmann equation, the expectation value the Hamiltonian (9) to be computed, in a quantum state under consideration. Therefore (9) becomes:

$$\langle \hat{H}_m \rangle = \frac{1}{2S^3} \langle \hat{\pi}^2 \rangle + \frac{m^2 S^3}{2} \langle \hat{\phi}^2 \rangle \ . \tag{24}$$

The eigenstates of the Hamiltonian are the Fock states:

$$\hat{a^\dagger}(t)\hat{a}(t)|n,\phi,t\rangle = n|n,\phi,t\rangle \ , \tag{25}$$

where $a^\dagger$,a are the creation and annihilation operators obeying boson commutation relations $[a,a^\dagger]=1$, the other combinations being zero. These can respectively be written as:

$$\begin{aligned} \hat{a}(t) &= \phi^*(t)\hat{\pi} - S^3\dot{\phi}^*(t)\hat{\phi}, \\ \hat{a^\dagger}(t) &= \phi(t)\hat{\pi} - S^3\dot{\phi}(t)\hat{\phi} \ . \end{aligned} \tag{26}$$

As an alternative to the n representation of the inflaton, next consider the inflaton in the coherent state formalism, therefore the semiclassical Einstein equation can be expressed in terms of the coherent state parameter.

From (11), (14) and (26), we get

$$\langle\hat{\pi}^2\rangle_{cs} = S^6\left[2(|\alpha|^2+\frac{1}{2})\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2\right] \tag{27}$$

and

$$\langle\hat{\phi}^2\rangle_{cs} = 2\left(|\alpha|^2+\frac{1}{2}\right)\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2 \ . \tag{28}$$

Substituting (27) and (28) in (24), the expectation value of the Hamiltonian obtained for the coherent state as:

$$\langle\hat{H}_m\rangle_{cs} = S^3\left[(|\alpha|^2+\frac{1}{2})(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) - \frac{1}{2}\alpha^{*2}[\dot{\phi}^{*2}+m^2\phi^{*2}] - \frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\right] \ . \tag{29}$$

In (29), ϕ and ϕ^* satisfy (22) and the Wronskian condition, given by:

$$S^3(t)\left(\dot{\phi}^*(t)\phi(t) - \phi^*(t)\dot{\phi}(t)\right) = i \ . \tag{30}$$

The Wronskian and the boundary conditions, fix the normalization constants of the two independent solutions.

Transform the solution in the following form

$$\phi(t) = \frac{1}{S^{\frac{3}{2}}}\psi(t), \tag{31}$$

thereby obtaining

$$\ddot{\psi}(t) + \left(m^2 - \frac{3}{4}\left(\frac{\dot{S}(t)}{S(t)}\right)^2 - \frac{3}{2}\frac{\ddot{S}(t)}{S(t)}\right)\psi(t) = 0 \ . \tag{32}$$

Next, focus on the oscillatory phase of the inflaton after inflation. In the parameter region satisfying the inequality

$$m^2 > \frac{3\dot{S}^2}{4S^2} + \frac{3\ddot{S}}{2S}, \tag{33}$$

the inflaton has an oscillatory solution of the form

$$\psi(t) = \frac{1}{\sqrt{2w(t)}} \exp(-i \int w(t) dt) \ . \tag{34}$$

With

$$w^2(t) = m^2 - \frac{3}{4}\left(\frac{\dot{S}(t)}{S(t)}\right)^2 - \frac{3}{2}\frac{\ddot{S}(t)}{S(t)} + \frac{3}{4}\left(\frac{\dot{w}(t)}{w(t)}\right)^2 - \frac{1}{2}\frac{\ddot{w}(t)}{w(t)} \ . \tag{35}$$

Next, consider the particle production of the inflaton, in coherent and squeezed states formalisms, in semiclassical theory of gravity. First, consider the Fock space which has a one parameter dependence on the cosmological time t. The number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t, t_0) = \langle 0, \phi, t_0 \mid \hat{N}(t) \mid 0, \phi, t_0 \rangle, \tag{36}$$

Here, $\hat{N}(t) = a^\dagger a$ and its expectation value can be calculated by using (26) as

$$\langle \hat{N}(t) \rangle = S^6 \dot{\phi}\dot{\phi}^* \langle \hat{\phi}^2 \rangle + \phi\phi^* \langle \hat{\pi}^2 \rangle - S^3 \phi\dot{\phi}^* \langle \hat{\pi}\hat{\phi} \rangle - S^3 \dot{\phi}\phi^* \langle \hat{\phi}\hat{\pi} \rangle \ . \tag{37}$$

$\langle \hat{\pi}^2 \rangle$, $\langle \hat{\pi}\hat{\phi} \rangle$, $\langle \hat{\phi}\hat{\pi} \rangle$ and $\langle \hat{\phi}^2 \rangle$ are respectively obtained as:

$$\begin{aligned} \langle \hat{\pi}^2 \rangle &= S^6 \dot{\phi}^* \dot{\phi}, \\ \langle \hat{\pi}\hat{\phi} \rangle &= S^3 \dot{\phi}\phi^*, \\ \langle \hat{\phi}\hat{\pi} \rangle &= S^3 \phi\dot{\phi}^*, \\ \langle \hat{\phi}^2 \rangle &= \phi^* \phi \ . \end{aligned} \tag{38}$$

Therefore, substituting (38), in (37), we get

$$N_0(t, t_0) = S^6 |\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2 \ . \tag{39}$$

Using the following approximation ansatzs

$$w_0(t) = m \tag{40}$$

and

$$S_0(t) = S_0 t^{\frac{2}{3}} \ , \tag{41}$$

and (31), the number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit mt_0, $mt > 1$ can be computed and is [7] given by:

$$\begin{aligned} N_0(t, t_0) &= \frac{1}{4w(t)w(t_0)}\left(\frac{S(t)}{S(t_0)}\right)^3 \Bigg[\frac{1}{4}\left(3\frac{\dot{S}(t)}{S(t)} - 3\frac{\dot{S}(t_0)}{S(t_0)} - \frac{\dot{w}(t)}{w(t)} + \frac{\dot{w}(t_0)}{w(t_0)}\right)^2 \\ &\quad + (w(t) - w(t_0))^2\Bigg] \\ &\simeq \frac{(t - t_0)^2}{4m^2 t_0^4} \ . \end{aligned} \tag{42}$$

The expectation values of $\langle \hat{\pi}^2 \rangle$, $\langle \hat{\pi}\hat{\phi} \rangle$, $\langle \hat{\phi}\hat{\pi} \rangle$ and $\langle \hat{\phi}^2 \rangle$ in the coherent state are obtained as:

$$\begin{aligned}
\langle \hat{\pi}^2 \rangle_{cs} &= S^6 \left[\left(2|\alpha|^2 + 1 \right) \dot{\phi}^*(t_0)\dot{\phi}(t_0) - \alpha^{*2}\dot{\phi}^{*2}(t_0) - \alpha^2 \dot{\phi}^2(t_0) \right] \ , \\
\langle \hat{\phi}^2 \rangle_{cs} &= \left(2|\alpha|^2 + 1 \right) \phi^*(t_0)\phi(t_0) - \alpha^{*2}\phi^{*2}(t_0) - \alpha^2 \phi^2(t_0) \ , \\
\langle \hat{\pi}\hat{\phi} \rangle_{cs} &= S^3 \left[|\alpha|^2 \dot{\phi}^*(t_0)\phi(t_0) + \left(|\alpha|^2 + 1 \right) \dot{\phi}(t_0)\phi^*(t_0) - \alpha^2 \dot{\phi}(t_0)\phi(t_0) - \alpha^2 \dot{\phi}^*(t_0)\phi(t_0) \right] , \\
\langle \hat{\phi}\hat{\pi} \rangle_{cs} &= S^3 \left[|\alpha|^2 \phi^*(t_0)\dot{\phi}(t_0) + \left(|\alpha|^2 + 1 \right) \phi(t_0)\dot{\phi}^*(t_0) - \alpha^2 \phi(t_0)\dot{\phi}(t_0) - \alpha^{*2} \phi(t_0)\dot{\phi}^*(t_0) \right] \ .
\end{aligned} \tag{43}$$

Substituting (43) in (37) and using (40) and (41), the number of particles at a later time t produced from the coherent state at the initial time t_0 is obtained as:

$$N_{cs} = (2|\alpha|^2 + 1)N_0(t, t_0) + |\alpha|^2 - S^6 \alpha^{*2} E - S^6 \alpha^2 F \ . \tag{44}$$

Here, $N_0(t, t_0)$ is given by (42) and

$$\begin{aligned}
E &= \phi(t)\phi^*(t)\dot{\phi}^{*2}(to) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0) - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}(t)\dot{\phi}^*(t)\phi^{*2}(t_0) \\
&= \frac{1}{4w(t)w(t_0)S^3(t)S^3(t_0)} \Bigg[\exp(2i \int w(t_0)dt_0 \Bigg(\mathcal{A}_2 - w^2(t_0) - iw(t_0) \\
&\quad \times \left(\frac{3\dot{S}(t_0)}{S(t_0)} + \frac{\dot{w}(t_0)}{w(t_0)} \right) + \mathcal{A}_1 + w^2(t) \Bigg) - 2\mathcal{C}_2 \left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)} \right] \Bigg] ,
\end{aligned} \tag{45}$$

$$\begin{aligned}
F &= \phi(t)\phi^*(t)\dot{\phi}^2(to) - \phi(t)\dot{\phi}^*(t)\dot{\phi}(t_0)\phi(t_0) - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}(t_0) + \dot{\phi}(t)\dot{\phi}^*(t)\phi^2(t_0) \ . \\
&= \frac{\exp(-2i \int w(t_0)dt_0}{4w(t)w(t_0)S^3(t)S^3(t_0)} \left[\left[\mathcal{A}_2 - w^2(t_0) + iw(t_0) \left(\frac{3\dot{S}(t_0)}{S(t_0)} + \frac{\dot{w}(t_0)}{w(t_0)} \right) \right] \right. \\
&\quad - \left[\frac{3\dot{S}(t_0)}{S(t_0)} + \frac{\dot{w}(t_0)}{w(t_0)} + 2iw(t_0) \right] \mathcal{C}_1 \\
&\quad \left. - \left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)} + 2iw(t) \right] \mathcal{C}_2 + \mathcal{A}_1 + w^2(t) \right] .
\end{aligned} \tag{46}$$

where,

$$\mathcal{A}_1 = \frac{1}{4} \left[3\frac{\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)} \right]^2 , \tag{47}$$

$$\mathcal{A}_2 = \frac{1}{4} \left[3\frac{\dot{S}(t_0)}{S(t_0)} + \frac{\dot{w}(t_0)}{w(t_0)} \right]^2 , \tag{48}$$

$$\mathcal{C}_1 = \frac{3}{4}\frac{\dot{S}(t)}{S(t)} + \frac{1}{4}\frac{\dot{w}(t)}{w(t)} - \frac{1}{2} iw(t) \ , \tag{49}$$

and

$$\mathcal{C}_2 = \frac{3}{4}\frac{\dot{S}(t_0)}{S(t_0)} + \frac{1}{4}\frac{\dot{w}(t_0)}{w(t_0)} - \frac{1}{2} iw(t_0). \tag{50}$$

Substituting (45) and (46) in (44), then using approximation ansatzs, the expression becomes:

$$
\begin{aligned}
N_{cs} = \quad & (2|\alpha|^2+1)N_0(t,t_0)+|\alpha|^2-\left(\frac{\alpha^2}{4m^2}\right)\left(\frac{t}{t_0}\right)^2 \\
& \times\left[\exp(2i\int(\theta-mt_0)\left(\frac{1}{t_0^2}-3m^2+\frac{2im}{t_0}-\frac{2}{tt_0}+im\frac{(t-t_0)}{tt_0}\right)\right. \\
& \left.+\exp(-2i\int(\theta-mt_0)\left(\frac{1}{t_0^2}-\frac{2im}{t_0}+\frac{1}{t^2}\right)-\exp(-2i\theta)\left(\frac{2}{tt_0}-\frac{2im}{t}\right)\right] .
\end{aligned} \tag{51}
$$

Therefore, the number of particles at a later time t produced from the coherent state at the initial time t_0, in the limit mt_0, $mt > 1$, is obtained by setting $\theta = mt_0 = 0$ and dropping the imaginary terms:

$$
N_{cs} \simeq \quad (2|\alpha|^2+1)N_0(t,t_0)+|\alpha|^2-\left(\frac{\alpha^2}{4m^2}\right)\left(\frac{t}{t_0}\right)^2\left[\frac{2}{t_0^2}+\frac{1}{t^2}-3m^2-\frac{4}{tt_0}\right] . \tag{52}
$$

Substituting $N_0(t,t_0)$ from (42) in (52),we get

$$
N_{cs} \simeq \quad (2|\alpha|^2+1)\frac{(t-t_0)^2}{4m^2t_0^4}+|\alpha|^2-\left(\frac{\alpha^2}{4m^2t_0^4}\right)\left[2t^2+t_0^2-3t_0^2t^2m^2-4tt_0\right] . \tag{53}
$$

Analogously, the number of particles at a later time t produced from squeezed vacuum state at the initial time t_0 can be obtained by using the following equations,

$$
\begin{aligned}
\langle\hat{\pi}^2\rangle_{svs} &= S^6\left[\left(2\sinh^2 r+1\right)\dot{\phi}^*(t_0)\dot{\phi}(t_0)+\sinh r\cosh r(e^{-i\varphi}\dot{\phi}^{*2}(t_0)+e^{i\varphi}\dot{\phi}^2(t_0))\right], \\
\langle\hat{\phi}^2\rangle_{svs} &= \left(2\sinh^2 r+1\right)\phi^*(t_0)\phi(t_0)+\sinh r\cosh r(e^{-i\varphi}\phi^{*2}(t_0)+e^{i\varphi}\phi^2(t_0)), \\
\langle\hat{\pi}\hat{\phi}\rangle_{svs} &= S^3\left[\sinh^2 r\dot{\phi}^*(t_0)\phi(t_0)+\cosh^2 r\dot{\phi}(t_0)\phi^*(t_0)+\right. \\
&\quad \left.\sinh r\cosh r(e^{-i\varphi}\dot{\phi}^*(t_0)\phi(t_0)+e^{i\varphi}\dot{\phi}(t_0)\phi(t_0))\right], \\
\langle\hat{\phi}\hat{\pi}\rangle_{svs} &= S^3\left[\sinh^2 r\phi^*(t_0)\dot{\phi}(t_0)+\cosh^2 r\phi(t_0)\dot{\phi}^*(t_0)\right. \\
&\quad \left.+\sinh r\cosh r(e^{-i\varphi}\phi(t_0)\dot{\phi}^*(t_0)+e^{i\varphi}\phi(t_0)\dot{\phi}(t_0))\right],
\end{aligned} \tag{54}
$$

and (54) in (37). The number of particles produced in squeezed vacuum state is then obtained as:

$$
N_{svs} = (2\sinh^2 r+1)N_0(t,t_0)+\sinh^2 r+S^6\sinh r\cosh r(e^{-i\varphi}E+e^{i\varphi}F) , \tag{55}
$$

where $N_0(t,t_0)$ is given by (42), E and F are respectively given by (45) and (46) .

By setting $\varphi = 2mt_0 = 0$ and dropping the imaginary terms , the above equations becomes:

$$
N_{svs} \simeq (2\sinh^2 r+1)\frac{(t-t_0)^2}{4m^2t_0^4}+\sinh^2 r+\frac{\sinh r\cosh r}{4m^2t_0^4}\left[2t^2+t_0^2-4tt_0-3m^2t_0^2t^2-2t_0t^2\right] . \tag{56}
$$

Similarly the number of particles at a later time t produced from the squeezed state at the initial time t_0 is obtained as:

$$\begin{aligned}
N_{ss} &= (2\sinh^2 r + 1 + 2|\alpha|^2)N_0(t,t_0) + (\sinh^2 r + |\alpha|^2) \\
&\quad + S^6 \sinh r \cosh r (e^{-i\varphi}E + e^{i\varphi}F) - S^6\alpha^{*2}E - S^6\alpha^2 F \ , \qquad (57) \\
&\simeq (2\sinh^2 r + 2|\alpha|^2 + 1)\frac{(t-t_0)^2}{4m^2t_0^4} + \sinh^2 r + |\alpha|^2 \\
&\quad + \frac{\sinh r \cosh r}{4m^2t_0^4}\left[2t^2 + t_0^2 - 4tt_0 - 3m^2t_0^2t^2 - 2t_0t^2\right] \\
&\quad - \left(\frac{\alpha^2}{4m^2t_0^4}\right)\left[2t^2 + t_0^2 - 4tt_0 - 3m^2t_0^2t^2\right] \ .
\end{aligned}$$

The above expression, when $r = 0$, equals the number of particles produced in the coherent state, and for $\alpha = 0$, it equals the number of particles produced in the squeezed vacuum state.

5 Comparison with classical gravity

Here, we briefly discuss some aspects of the inflaton in classical gravity with semiclassical gravity, in its oscillatory regime. Since coherent states are considered as the closest to the classical state, it is instructive to compare the result of this formalism with the classical gravity theory.

Consider the oscillatory phase of inflaton; the initial values of the inflaton can be incorporated with the amplitude and the phase of the real inflaton

$$\phi_r(t) = \frac{\mathcal{B}_0}{S^{\frac{3}{2}}(t)\sqrt{w(t)}}\sin\left(\int w(t)dt + \delta\right) \ , \qquad (58)$$

where $\mathcal{B}_0$ is the amplitude of the classical inflaton.

The Hamiltonian for the classical inflaton is given by

$$H_m = \frac{1}{2S^3}\pi_{\phi_r}^2 + \frac{m^2S^3}{2}\phi_r^2 \ , \qquad (59)$$

$\pi_{\phi_r}^2$ and ϕ_r^2 can be evaluated and these values are substituted in (59). The classical energy density for the inflaton in classical gravity becomes

$$\begin{aligned}
H_m &= \frac{\mathcal{B}_0^2}{2}\frac{1}{2w}\left[m^2 + w^2 + \frac{1}{4}\left(\frac{\dot{w}}{w} + 3\frac{\dot{S}}{S}\right)^2 + \left(w^2 - \frac{1}{4}\left(\frac{\dot{w}}{w} + 3\frac{\dot{S}}{S}\right)^2\right)\right. \\
&\quad \times \cos\left(2\int w(t)dt + \delta\right) - \left(\frac{\dot{w}}{w} + 3\frac{\dot{S}}{S}\right)\sin 2\left(\int w(t)dt + \delta\right) \\
&\quad \left. - m^2\cos 2\left(\int w(t)dt\right)\right] \ . \qquad (60)
\end{aligned}$$

The result of the above Hamiltonian (60) shows the oscillatory behavior of the classical energy density. The oscillating terms determine, in a significant way, the evolution of the geometric invariants through the higher derivatives of the scale factor S(t).

The time average over several oscillations of the Hamiltonian (60) can be computed by assuming that the expansion of the universe can be neglected during each period of coherent oscillation and is obtained [7] as

$$\langle H_m\rangle_{tavg} \quad = \quad \frac{\mathcal{B}_0^2}{2}\frac{1}{2w}\left[\frac{1}{4}\left(\frac{\dot{w}}{w}+3\frac{\dot{S}}{S}\right)^2+m^2+w^2\right] \; . \tag{61}$$

Next goal is to compare the above classical result with that of the semiclassical energy of the inflaton in the coherent state. Hence, consider the Hamiltonian of the inflaton in coherent state in (29). Substituting (31) in (29), the energy density in the coherent state becomes:

$$\begin{aligned}\langle \hat{H}_m\rangle_{cs} = \quad & \frac{1}{2w(t)}\left[(|\alpha|^2+\frac{1}{2})(\mathcal{A}_1+w^2(t)+m^2)\right. \\ & -\frac{1}{2}\alpha^{*2}\exp(2i\int w(t)dt\left(\mathcal{A}_1-w^2(t)-iw(t)\left[3\frac{\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right) \\ & \left.-\frac{1}{2}\alpha^2\exp(-2i\int w(t)dt\left(\mathcal{A}_1-w^2(t)+iw(t)\left[3\frac{\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)\right]^{\frac{1}{3}}\end{aligned} \tag{62}$$

where $\mathcal{A}_1$ is given by (47) . As in the case of the classical energy density, the time average over several oscillations of the energy density of the inflaton in coherent state can be computed by assuming that the expansion of the universe can be neglected during each period, and is obtained as

$$\langle \hat{H}_m\rangle_{cs-tavg} = \quad \left(|\alpha|^2+\frac{1}{2}\right)\frac{1}{2w(t)}\left[\frac{1}{4}\left(\frac{\dot{w}}{w}+3\frac{\dot{S}}{S}\right)^2+w^2(t)+m^2\right] \; . \tag{63}$$

Comparing (61) with (63), we obtain the classical amplitude:

$$\mathcal{B}_0=\sqrt{2|\alpha|^2+1}. \tag{64}$$

6 Discussions and conclusions

In this paper, we studied particle production of the coherently oscillating inflaton, after the inflation, in coherent and squeezed states formalisms, in the frame work of semiclassical theory of gravity. The number of particles at a later time t, produced from the coherent state, at the initial time t_0, in the limit mt_0, $mt > 1$ calculated. It show, the particle production

depends on the coherent state parameter. The particle creation in squeezed state in the limit $mt_0 > mt > 1$ is also computed , it is found that the particle production depending on the associated squeezing parameter. Similarly the number of particles produced in the squeezed state also computed. It is observed that, when $r = 0$, the result matches with the number of particles produced in the coherent state and when $\alpha = 0$, the result fits with the number of particles produced in the squeezed vacuum state.

Quantum effects can play a significant role in the oscillatory phase of the inflaton after inflation in the event that, coherent and squeezed states are possible quantum states of the inflaton in that regime, the particle production can be affected the coherent oscillations of the inflaton after the inflation. This is became the fact that, the inflaton after inflation can not execute coherent oscillation for a sufficiently long period of time, since it will suffer from an instability due to the particle production. The particle production in coherent state, squeezed vacuum state and squeezed state also depend on the duration of the coherent oscillations of the inflaton.

In this paper, we have also studied the inflaton in classical gravity and compare the results with the inflaton in semiclassical gravity by representing it in coherent state formalism, since the coherent states are considered as the closest to the classical states. From the comparative study of the energy density of the inflaton in coherent state with the classical energy density, it is found that the energy density in coherent state also exhibit oscillatory behavior apart from the quantum correction. This could be due to the inherent quantum nature of the coherent state. Hence, the semiclassical energy density and classical energy density may differ because of the non-oscillatory nature of the quantum correction of the energy density in coherent state formalism. It can also be concluded that, if the expansion of the universe can be neglected during each period of coherent oscillation, then the time-averaged classical energy density, in the leading order, is the same as that given by the semiclassical gravity in the coherent state, if one identifies the classical amplitude with $\mathcal{B}_0 = \sqrt{2|\alpha|^2 + 1}$. A plot for coherent state parameter verses $\mathcal{B}_0$ is presented in Fig.1.

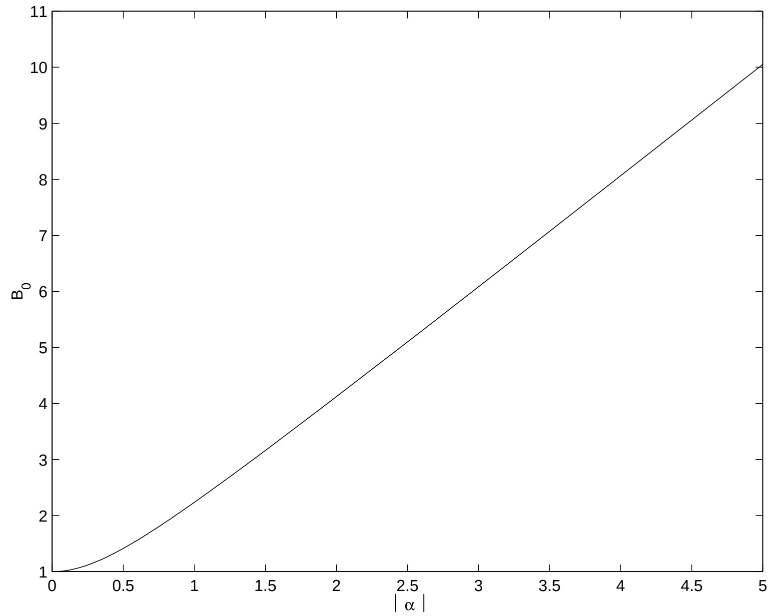

Fig.1.Plot for $|\alpha|$ with $\mathcal{B}_0$.

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References

[1] A. H. Guth, *Phys.Rev.* **D23**, 347 (1981).

[2] A. D. Linde, *Phys. Lett.***B 108**, 389 (1982).

[3] A. D. Linde, *Phys.Lett.*.**B129**, 177 (1983).

[4] A. Albrecht and P. J. Steinhardt, *Phys.Rev.Lett.***48**,1220 (1982).

[5] R. Brandenberger, *Rev.Mod.Phys.***57**, 1 (1985).

[6] B. Dongsu Bak et.al., *Phys.Rev.***D59**, 027301 (1998).

[7] S. P. Kim and D. N. Page, *J.Kor.Phys.Soc.***35**, S660 (1999).

[8] A. Guth and Pi. S-Y, *Phys.Rev.***D32**, 1899 (1995).

[9] A. Linde, D. Linde and A Mezhlumian, *Phys.Rev.***D49**, 1783 (1994).

[10] S. Habib, *Phys.Rev.***D46**, 2408 (1992).

[11] B. K. Berger, *Phys.Rev.***D23**, 1250 (1981).

[12] L. P. Grishchuk and Y. V. Sidorov, *Phys.Rev.***D42**, 3413 (1993).

[13] R. Brandenberger, V. Mukhanov and T. Prokopec, *Phys.Rev.Lett.***69**, 3606 (1992).

[14] R. Brandenberger, V. Mukhanov and T.Prokopec, *Phys.Rev.***D48**, 2443 (1993).

[15] C. Kuo and L. H. Ford, *Phys.Rev.***D46**, 4510 (1993).

[16] A. L. Matacz, P. C. W. Davies and A. C. Ottewill, *Phys.Rev.***D47**, 1557 (1993).

[17] A. Albrecht et.al., *Phys.Rev.***D50**, 4807 (1994).

[18] M. Gasperini and M. Giovanni, *Phys.Rev.***D50**, 4807 (1994).

[19] B. L. Hu, G. Kang and A. L. Matacz, *Int.J.Mod.Phys.***A9**, 991 (1994).

[20] P. K. Suresh, V. C. Kuriakose and K. Babu Joseph, *Int.J.Mod.Phys.***D6**, 771 (1995).

[21] P. K. Suresh and V. C. Kuriakose, *Mod.Phys.Lett.***A13**, 165 (1998).

[22] N. D. Birrell and P. C. W. Davies, *Quantum Field Theory in Curved Space* (Cambridge University Press, 1982).

[23] J. R. Klauder and B. S. Skagersterm, *Coherent States* (WorldScientific, 1985).

[24] C. M. Caves and D. F. Walls, *Phys. Rev.Lett.***57**, 2164 (1980).

[25] B. L. Schumaker, *Phys.Rep.* **135**, 317 (1986).